HW10

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To prove LTFS is NP-Complete, we need to:

1. Show that LTFS is in NP
2. Give a mapping reduction from an NP-Complete problem to LTFS
3. If we are given a subset of size >= k of graph G, we can enumerate each triple in the point set of the subset, for each triple, we can judge that if the triple has at least one absent edge. If the triple meets the restriction then continue, if not the subset is not a triangle free subset. And finally we could verify that it is a triangle free subset or not in polynomial time. So LTFS is in NP.
4. We give a mapping reduction from Independent Set(IS) to LTFS.

To reduce the IS to LTFS, we can construct a mapping from Independent Set to LTFS. We can construct a mapping from an instance of IS(G, k) to an instance of LTFS(G’, k’) if (G’, k’) has the same value as (G, k). First suppose that we are given an instance (G, k) of IS where G = (V, E) (V = n; E = m), we can create a new vertex we for each e = (u, v) ∈ E and call the set of these vertices W. For each new vertex we ∈ Ve where e = (u, v), create two new edges (we, u) and (we, v) and call this edge set Ew. Then we Set G’ to be the graph G with the added vertices and edges onto it, i.e. G = (V∪W, E∪Ew). Set k’ = m + k.

So the reduction substitute each edge e = (u, v) in G by a triangle {u, we, v}.

Now we show that if G’ has a triangle-free subset of size at least k’ = m + k then G has an independent set of size at least k.

Pick out an independent set S of size k in V. Let S’ = S ∪ W . We know that |S’|= m + k. Then S’ is triangle-free. Indeed, in G’ the set of vertices S is triangle-free since no two of them are connected by any edge; otherwise S is not independent.

Moreover, the addition of any vertex in W will not create a triangle. For each we ∈ W with e = (u, v). Since S is independent, u and v cannot be both in S. If u and v are both not in S, then adding we to S’ will not create a triangle since it is not connected to any other vertex in S’. If u or v is in S, then adding we won’t create a triangle because a triangle containing we must have both u and v in it.

Then we argue that if G’ has a triangle-free subset of size at least k’ = m + k, then G has an independent set of size at least k.

Pick any triangle-free subset S’ of size m + k from G’. If S’ contains the whole set W , writing S’= S ∪Wwith S ⊆ V , then S must be an independent set of G. The reason is that no two vertices u and v in S can share an edge e; otherwise since we, u, and v are all in S’ and they form a triangle, S’ is not triangle-free, a contradiction. If S’ ∩ W ≠ W, we argue that we can always find another triangle-free subset S’’ containing the entire W with S’’ = S∪W , S ⊆ V such that |S’’| ≥ m + k. If this is true, then as argued above, S is an independent set of size k in G.

With S’ in our hand, for each vertex we∈ W with e = (u, v) but we is not exist in S’, do the following:

• If at least one of u and v is not exist in S’, add we to S’.

• If u and v are both in S’, remove u from S’ and add we to S’.

For all of the possibilities above, we put we into S’ without creating a triangle in S’ and without decreasing the size of S’. After we do this for every vertex we∈W, we get S’’ with the entire W is in it and |S’’| ≥ m + k.

So on the above we can see that we can make a reduction from an exist NP-Complete problem to problem of LTFS in polynomial time, then the problem of LTFS is NP-Complete.